

Exercise 2.3 — Using the binomial probability function

Q1 a) Use the binomial probability function with $n = 10$ and $p = 0.14$.

$$\begin{aligned}\text{(i)} \quad P(X = 2) &= \binom{10}{2} \times 0.14^2 \times (1 - 0.14)^{10-2} \\ &= \frac{10!}{2!8!} \times 0.14^2 \times 0.86^8 = 0.264 \text{ (to 3 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad P(X = 4) &= \binom{10}{4} \times 0.14^4 \times (1 - 0.14)^{10-4} \\ &= \frac{10!}{4!6!} \times 0.14^4 \times 0.86^6 = 0.0326 \text{ (to 3 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad P(X = 5) &= \binom{10}{5} \times 0.14^5 \times (1 - 0.14)^{10-5} \\ &= \frac{10!}{5!5!} \times 0.14^5 \times 0.86^5 = 0.00638 \text{ (to 3 s.f.)}\end{aligned}$$

b) Use the binomial probability function with $n = 8$ and $p = 0.27$.

$$\begin{aligned}\text{(i)} \quad P(X = 3) &= \binom{8}{3} \times 0.27^3 \times (1 - 0.27)^{8-3} \\ &= \frac{8!}{3!5!} \times 0.27^3 \times 0.73^5 = 0.229 \text{ (to 3 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad P(X = 5) &= \binom{8}{5} \times 0.27^5 \times (1 - 0.27)^{8-5} \\ &= \frac{8!}{5!3!} \times 0.27^5 \times 0.73^3 = 0.0313 \text{ (to 3 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad P(X = 7) &= \binom{8}{7} \times 0.27^7 \times (1 - 0.27)^{8-7} \\ &= \frac{8!}{7!1!} \times 0.27^7 \times 0.73^1 = 0.000611 \text{ (to 3 s.f.)}\end{aligned}$$

Q2 a) Use the binomial probability function with $n = 20$ and $p = 0.16$.

$$(i) P(X < 2) = P(X = 0) + P(X = 1)$$

$$= \frac{20!}{0!20!} \times 0.16^0 \times 0.84^{20} + \frac{20!}{1!19!} \times 0.16^1 \times 0.84^{19}$$

$$= 0.03059... + 0.11653... = 0.147 \text{ (to 3 s.f.)}$$

$$(ii) P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$= 0.03059... + 0.11653... + \frac{20!}{2!18!} \times 0.16^2 \times 0.84^{18}$$

$$+ \frac{20!}{3!17!} \times 0.16^3 \times 0.84^{17}$$

$$= 0.03059... + 0.11653... + 0.21087... + 0.24099...$$

$$= 0.599 \text{ (to 3 s.f.)}$$

$$(iii) P(1 < X \leq 4) = P(X = 2) + P(X = 3) + P(X = 4)$$

$$= 0.21087... + 0.24099... + \frac{20!}{4!16!} \times 0.16^4 \times 0.84^{16}$$

$$= 0.21087... + 0.24099... + 0.19509...$$

$$= 0.647 \text{ (to 3 s.f.)}$$

b) Use the binomial probability function with $n = 30$ and $p = 0.88$.

$$(i) P(X > 28) = P(X = 29) + P(X = 30)$$

$$= \frac{30!}{29!1!} \times 0.88^{29} \times 0.12^1 + \frac{30!}{30!0!} \times 0.88^{30} \times 0.12^0$$

$$= 0.088369... + 0.021601...$$

$$= 0.110 \text{ (to 3 s.f.)}$$

$$(ii) P(25 < X < 28) = P(X = 26) + P(X = 27)$$

$$= \frac{30!}{26!4!} \times 0.88^{26} \times 0.12^4 + \frac{30!}{27!3!} \times 0.88^{27} \times 0.12^3$$

$$= 0.204693... + 0.222383...$$

$$= 0.427 \text{ (to 3 s.f.)}$$

$$(iii) P(X \geq 27) = P(X = 27) + P(X = 28)$$

$$+ P(X = 29) + P(X = 30)$$

$$= 0.222383... + \frac{30!}{28!2!} \times 0.88^{28} \times 0.12^2$$

$$+ 0.088369... + 0.021601...$$

$$= 0.222383... + 0.174729... + 0.088369...$$

$$+ 0.021601...$$

$$= 0.507 \text{ (to 3 s.f.)}$$

Q3 a) Use the binomial probability function

with $n = 5$ and $p = \frac{1}{2}$.

$$\begin{aligned}\text{(i)} \quad P(X \leq 4) &= 1 - P(X > 4) = 1 - P(X = 5) \\ &= 1 - \frac{5!}{5!0!} \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^0 = 1 - 0.03125 \\ &= 0.969 \text{ (to 3 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad P(X > 1) &= 1 - P(X \leq 1) = 1 - P(X = 0) - P(X = 1) \\ &= 1 - \frac{5!}{0!5!} \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^5 - \frac{5!}{1!4!} \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^4 \\ &= 1 - 0.03125 - 0.15625 = 0.813 \text{ (to 3 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad P(1 \leq X \leq 4) &= 1 - P(X = 0) - P(X = 5) \\ &= 1 - \frac{5!}{0!5!} \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^5 - \frac{5!}{5!0!} \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^0 \\ &= 1 - 0.03125 - 0.03125 = 0.938 \text{ (to 3 s.f.)}\end{aligned}$$

b) Use the binomial probability function

with $n = 8$ and $p = \frac{2}{3}$.

$$\begin{aligned}\text{(i)} \quad P(X < 7) &= 1 - P(X \geq 7) = 1 - P(X = 7) - P(X = 8) \\ &= 1 - \frac{8!}{7!1!} \times \left(\frac{2}{3}\right)^7 \times \left(\frac{1}{3}\right)^1 - \frac{8!}{8!0!} \times \left(\frac{2}{3}\right)^8 \times \left(\frac{1}{3}\right)^0 \\ &= 1 - 0.156073... - 0.039018... = 0.805 \text{ (to 3 s.f.)}\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad P(X \geq 2) &= 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1) \\ &= 1 - \frac{8!}{0!8!} \times \left(\frac{2}{3}\right)^0 \times \left(\frac{1}{3}\right)^8 - \frac{8!}{1!7!} \times \left(\frac{2}{3}\right)^1 \times \left(\frac{1}{3}\right)^7 \\ &= 1 - 0.00015241... - 0.00243865... \\ &= 0.997 \text{ (to 3 s.f.)}\end{aligned}$$

$$\text{(iii)} \quad P(0 \leq X \leq 8) = 1$$

This must be 1, as X can only take values from 0 to 8.

Q4 $n = 5$ and $p = P(\text{roll a six}) = \frac{1}{6}$, so

$$P(2 \text{ sixes}) = \binom{5}{2} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^3 = 0.161 \text{ (to 3 s.f.)}$$

Q5 a) There are 12 answers, which are either 'correct' or 'incorrect'. The student guesses at random so the questions are answered independently of each other and the probability of a correct answer is $\frac{1}{3}$.

So $X \sim B(12, \frac{1}{3})$.

$$\begin{aligned}
 \text{b) } P(X < 3) &= P(X = 0) + P(X = 1) + P(X = 2) \\
 &= \frac{12!}{0!12!} \times \left(\frac{1}{3}\right)^0 \times \left(\frac{2}{3}\right)^{12} + \frac{12!}{1!11!} \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^{11} \\
 &\quad + \frac{12!}{2!10!} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^{10} \\
 &= 0.00770... + 0.04624... + 0.12717... \\
 &= 0.181 \text{ (to 3 s.f.)}
 \end{aligned}$$

- Q6** Let X represent the number of defective items.
 Then $X \sim B(15, 0.05)$, and you need to find $P(1 \leq X \leq 3)$.
 $P(1 \leq X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3)$
 $= \frac{15!}{1!14!} \times 0.05^1 \times 0.95^{14} + \frac{15!}{2!13!} \times 0.05^2 \times 0.95^{13}$
 $\quad + \frac{15!}{3!12!} \times 0.05^3 \times 0.95^{12}$
 $= 0.36575... + 0.13475... + 0.03073... = 0.531 \text{ (to 3 s.f.)}$

- Q7** a) Let the random variable X represent the number of 'treble-20's the player gets in a set of 3 darts.
 Then $X \sim B(3, 0.75)$.
 $P(X \geq 2) = P(X = 2) + P(X = 3)$
 $P(X = 2) = \binom{3}{2} \times 0.75^2 \times (1 - 0.75) = 0.421875$
 $P(X = 3) = \binom{3}{3} \times 0.75^3 \times (1 - 0.75)^0 = 0.421875$
 So $P(X \geq 2) = 0.421875 + 0.421875$
 $= 0.84375 = 0.844 \text{ (to 3 s.f.)}$

- b) Now let X represent the number of 'treble-20's the player scores with 30 darts. Then $X \sim B(30, 0.75)$.
 You need to find $P(X \geq 26)$.
 $P(X \geq 26) = P(X = 26) + P(X = 27) + P(X = 28)$
 $\quad + P(X = 29) + P(X = 30)$
 $= 0.06042... + 0.02685... + 0.00863...$
 $\quad + 0.00178... + 0.00017... = 0.0979 \text{ (to 3 s.f.)}$

Exercise 3.1 — Using a calculator to find probabilities

- Q1**
- a) $P(X \leq 2) = 0.526$ (3 s.f.)
 - b) $P(X \leq 6) = 0.996$ (3 s.f.)
 - c) $P(X \leq 9) = 1.00$ (3 s.f.)
 - d) $P(X < 5) = P(X \leq 4) = 0.922$ (3 s.f.)
 - e) $P(X < 4) = P(X \leq 3) = 0.776$ (3 s.f.)
 - f) $P(X < 6) = P(X \leq 5) = 0.980$ (3 s.f.)

- Q2**
- a) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0.0905... = 0.909$ (3 s.f.)
 - b) $P(X > 6) = 1 - P(X \leq 6) = 1 - 0.6098... = 0.390$ (3 s.f.)
 - c) $P(X > 10) = 1 - P(X \leq 10) = 1 - 0.990652... = 0.00935$ (3 s.f.)
 - d) $P(X \geq 5) = 1 - P(X < 5) = 1 - P(X \leq 4) = 1 - 0.2172... = 0.783$ (3 s.f.)
 - e) $P(X \geq 3) = 1 - P(X < 3) = 1 - P(X \leq 2) = 1 - 0.0271... = 0.973$ (3 s.f.)
 - f) $P(X \geq 13) = 1 - P(X < 13) = 1 - P(X \leq 12) = 1 - 0.9997210... = 0.000279$ (3 s.f.)

- Q3**
- a) $P(X = 7) = 0.184$ (3 s.f.)
If your calculator doesn't have a binomial pdf, you can either work these out using the binomial probability formula, or calculate $P(X \leq 7) - P(X \leq 6)$ with the binomial cdf.
 - b) $P(X = 12) = 0.0136$ (3 s.f.)
 - c) $P(2 < X \leq 4) = P(X \leq 4) - P(X \leq 2) = 0.11819... - 0.01211... = 0.106$ (3 s.f.)
 - d) $P(10 < X \leq 15) = P(X \leq 15) - P(X \leq 10) = 0.99995... - 0.94683... = 0.0531$ (3 s.f.)

- e) $P(7 \leq X \leq 10) = P(X \leq 10) - P(X \leq 6)$
 $= 0.94683... - 0.41662... = 0.530$ (3 s.f.)
- f) $P(3 \leq X < 11) = P(X \leq 10) - P(X \leq 2)$
 $= 0.94683... - 0.01211... = 0.935$ (3 s.f.)

- Q4** a) $P(X \geq 17) = 1 - P(X < 17) = 1 - P(X \leq 16)$
 $= 1 - 0.04677... = 0.953$ (3 s.f.)
- b) $P(X \geq 20) = 1 - P(X < 20) = 1 - P(X \leq 19)$
 $= 1 - 0.38331... = 0.617$ (3 s.f.)
- c) $P(X > 14) = 1 - P(X \leq 14)$
 $= 1 - 0.00555... = 0.994$ (3 s.f.)
- d) $P(X = 21) = 0.187$ (3 s.f.)
- e) $P(14 \leq X < 17) = P(X \leq 16) - P(X \leq 13)$
 $= 0.04677... - 0.00154... = 0.0452$ (3 s.f.)
- f) $P(12 < X \leq 18) = P(X \leq 18) - P(X \leq 12)$
 $= 0.21996... - 0.00036... = 0.220$ (3 s.f.)

- Q5** Let X represent the number of children with green eyes.
 Then $X \sim B(30, 0.18)$.
 $P(X < 10) = P(X \leq 9) = 0.96768... = 0.968$ (3 s.f.)

- Q6** Let X represent the number of faulty items.
 Then $X \sim B(25, 0.05)$.
 $P(X < 6) = P(X \leq 5) = 0.99878... = 0.999$ (3 s.f.)